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$N = 2$ superconformal description of superstring in Ramond-Ramond plane wave backgrounds

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Abstract: Using the U(4) formalism developed ten years ago, the worldsheet action for the superstring in Ramond-Ramond plane wave backgrounds is expressed in a manifestly $N = (2, 2)$ superconformally invariant manner. This simplifies the construction of consistent Ramond-Ramond plane wave backgrounds and eliminates the problems associated with light-cone interaction point operators.

Keywords: Superstrings and Heterotic Strings, Penrose limit and pp-wave background.

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1. Introduction

Although the construction of worldsheet actions for the superstring in Neveu-Schwarz backgrounds is well understood using the Ramond-Neveu-Schwarz (RNS) formalism, the worldsheet action for the superstring in Ramond-Ramond (RR) backgrounds has been less studied. Because of the important role of Ramond-Ramond backgrounds in the AdS/CFT conjectures [1], these worldsheet actions might be very useful for studying aspects of these conjectures.

When the background allows a light-cone gauge choice, the most straightforward method for constructing the superstring action in Ramond-Ramond backgrounds is to use the light-cone Green-Schwarz (GS) formalism [2]. This light-cone formalism is extremely useful for computing the physical spectrum in a given background, however, it is difficult to use for computing scattering amplitudes or for determining consistency conditions on superstring backgrounds. Although the light-cone RNS formalism suffers from similar problems, in the RNS case there exists an $N = 1$ superconformally invariant description of the superstring which can replace the light-cone description (at least when RR fields are zero).

Over ten years ago, an $N = 2$ superconformally invariant description of the superstring was proposed [3, 4] which reduces in light-cone gauge to the light-cone GS description. In addition to the light-cone transverse variables which are combined into bosonic worldsheet $N = 2$ superfields, this superconformally invariant description also contains fermionic worldsheet $N = 2$ superfields which describe the longtitudinal variables. In a flat background, the action is quadratic and contains manifest U(4) Lorentz invariance as well as twenty manifest spacetime supersymmetries. This U(4) formalism can be obtained by gauge-fixing a "doubly-supersymmetric" action [5, 6, 3] where κ-symmetry is replaced by worldsheet supersymmetry, and is related to the RNS formalism by a field redefinition which maps the fermionic $N = 2$ superconformal generators to the twisted BRST current and b ghost in the RNS formalism [7]. Recently, this U(4) formalism has also been related to the pure spinor formalism [8] for the superstring via different gauge fixings [9] of the doubly-supersymmetric action.

In the pure spinor formalism of the superstring [8], all SO(9,1) super-Poincaré invariance is manifest but the original $N = 2$ worldsheet superconformal invariance is hidden. Although one can use the pure spinor formalism to describe any consistent background of the superstring, for describing Ramond-Ramond plane wave backgrounds in which Lorentz invariance is already broken, it is more advantageous to use the U(4) formalism so that the $N = 2$ worldsheet supersymmetry is manifest. For example, the conformally invariant action for the maximally supersymmetric Ramond-Ramond plane wave background [10] will be much simpler using the U(4) formalism than using the pure spinor formalism [11] . As will be shown here, the U(4) formalism is also extremely useful for determining consistency conditions for Ramond-Ramond plane wave backgrounds with fewer numbers of supersymmetries [12]. Furthermore, since this formalism does not require light-cone interaction point operators, it does not suffer from contact term problems and may also be useful for computing scattering amplitudes in these backgrounds.[1]

In section 2 of this paper, the U(4) formalism and its relation to the light-cone GS formalism will be reviewed. In section 3, $N = (2,2)$ superconformally invariant actions will be constructed for the superstring in plane wave Ramond-Ramond backgrounds which preserve either two or four spacetime supersymmetries, and will be proven to be exact superconformal field theories to all perturbative orders in α'. Such Ramond-Ramond backgrounds are naturally described by real or holomorphic superpotentials [12] and may be useful for studying various aspects of AdS/CFT conjectures. And in the Appendix, the relation between $N = 2$ superconformal invariance and the light-cone GS interaction point operator will be discussed.

2. Review of U(4) formalism

2.1 Flat background

In a flat background, the Type IIB GS action in light-cone gauge can be written as [14]

$$S = \int d^2z \left(\partial x^{+\bar{l}} \bar{\partial} x^{-l} + s^{-\bar{l}} \bar{\partial} s^{+l} + \bar{s}^{-\bar{l}} \partial \bar{s}^{+l} \right), \tag{2.1}$$

where SO(8) has been broken to SU(4) $\times$ U(1) such that the SO(8) vector splits into $(x^{+\bar{l}}, x^{-l})$ and the SO(8) chiral spinor splits into $(s^{-\bar{l}}, s^{+l})$ for $l = 1$ to 4. Note that under SU(4) $\times$ U(1), $x^{+\bar{l}}$ and x^{-l} transform as $(\bar{4}, +1)$ and $(4, -1)$ representations, whereas $s^{-\bar{l}}$ and s^{+l} transform as $(\bar{4}, -1)$ and $(4, +1)$ representations. The action of (2.1) can easily be written in $N = 2$ supersymmetric notation as

$$S = \int d^2z \int d^2\kappa^+ \int d^2\kappa^- \, X^{+\bar{l}} X^{-l}, \tag{2.2}$$

[1] In [4] and [13], multiloop scattering amplitudes were computed using the U(4) formalism and a proof was claimed for their finiteness. This proof was incorrect because of subtleties coming from unphysical poles in multiloop correlation functions of chiral bosons. However, the techniques developed in these papers can be used for computing tree and one-loop scattering amplitudes where such subtleties are absent.

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where $X^{+\bar{l}}$ and X^{-l} are chiral and antichiral superfields satisfying

$$D_- X^{+\bar{l}} = \bar{D}_- X^{+\bar{l}} = 0\,, \qquad D_+ X^{-l} = \bar{D}_+ X^{-l} = 0\,, \tag{2.3}$$

$$\begin{aligned} D_- &= \frac{\partial}{\partial \kappa^-} + \kappa^+ \partial_z\,, \qquad D_+ = \frac{\partial}{\partial \kappa^+} + \kappa^- \partial_z\,, \\ \bar{D}_- &= \frac{\partial}{\partial \bar{\kappa}^-} + \bar{\kappa}^+ \bar{\partial}_{\bar{z}}\,, \qquad \bar{D}_+ = \frac{\partial}{\partial \bar{\kappa}^+} + \bar{\kappa}^- \bar{\partial}_{\bar{z}}\,, \end{aligned} \tag{2.4}$$

$$X^{+\bar{l}} = x^{+\bar{l}} + \kappa^+ s^{-\bar{l}} + \bar{\kappa}^+ \bar{s}^{-\bar{l}} + \kappa^+ \bar{\kappa}^+ h^{-\bar{l}}, \quad X^{-l} = x^{-l} + \kappa^- s^{+l} + \bar{\kappa}^- \bar{s}^{+l} + \kappa^- \bar{\kappa}^- h^{+l}\,, \tag{2.5}$$

and $(h^{-\bar{l}}, h^{+l})$ are auxiliary fields.

To construct a conformally invariant action which reduces in light-cone gauge to (2.1), it is useful to recall the construction of conformally invariant actions for the bosonic and RNS string. In bosonic string theory, we can find a map from the complex plane to the light-cone string diagram $\rho(z)$ which is a conformal transformation. To construct a conformally invariant action for the bosonic string, the real part of this conformal map $\rho(z) + \bar{\rho}(\bar{z})$ is promoted to the target-space light-cone variable $x^+(z, \bar{z})$. A similar procedure can be performed for RNS string theory, however, in this case the light-cone string diagram is mapped to an $N = 1$ complex superplane using the $N = 1$ superconformal map $[\rho(z, \kappa), \xi(z, \kappa)]$ where $[\rho, \xi]$ parameterize the string diagram and $[z, \kappa]$ parameterize the $N = 1$ superplane. As reviewed in the Appendix, the use of an $N = 1$ superconformal map is crucial for obtaining the appropriate light-cone RNS interaction point operator. To construct an $N = 1$ superconformally invariant action, one promotes $\rho(z, \kappa) + \bar{\rho}(\bar{z}, \bar{\kappa})$ to the worldsheet $N = 1$ superfield X^+. Since the map is

$$N = 1$$

superconformal, $\xi(z, \kappa)$ is determined by $\rho(z, \kappa)$, so X^+ completely determines the $N = 1$ superconformal map.

It turns out that to obtain a conformally invariant description of the GS superstring, one needs to use an $N = 2$ superconformal transformation

$$\left[\rho(z, \kappa^+, \kappa^-), \xi^+(z, \kappa^+), \xi^-(z, \kappa^-)\right]$$

to map the $N = 2$ complex superplane $[z, \kappa^+, \kappa^-]$ to the string diagram $[\rho, \xi^+, \xi^-]$. In fact, as was shown in [3] and reviewed in the Appendix, precisely such an $N = 2$ superconformal map is required for obtaining the correct light-cone GS interaction point operator. However, unlike $N = 1$ superconformal transformations, the $\xi^+(z, \kappa^+)$ and $\xi^-(z, \kappa^-)$ parameters in an $N = 2$ superconformal map are not uniquely determined by $\rho(z, \kappa^+, \kappa^-)$ because of the possibility of performing U(1) transformations. For this reason, instead of promoting one variable to a superfield we actually need to promote two. We promote $\xi^+(z, \kappa^+)$ and $\xi^-(z, \kappa^-)$ to worldsheet superfields which will be called Θ^+ and Θ^-. For the Type II superstring, one also has the barred fermions $\bar{\xi}^+(\bar{z}, \bar{\kappa}^+)$ and $\bar{\xi}^-(\bar{z}, \bar{\kappa}^-)$ which will be promoted to $\bar{\Theta}^+$ and $\bar{\Theta}^-$.

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These fermionic target-space variables will be defined as $N = 2$ chiral and antichiral worldsheet superfields satisfying [3, 4]

$$D_-\Theta^+ = \bar{D}_-\Theta^+ = D_-\bar{\Theta}^+ = \bar{D}_-\bar{\Theta}^+ = 0\,, \qquad D_+\Theta^- = \bar{D}_+\Theta^- = D_+\bar{\Theta}^- = \bar{D}_+\bar{\Theta}^- = 0\,. \tag{2.6}$$

The fact that the map $[z,\kappa^-,\kappa^-] \to [\rho,\xi^+,\xi^-]$ is $N = 2$ superconformal then determines $\rho(z,\kappa^+,\kappa^-)$ which in turn determines X^+. More explicitly, we have that ρ is related to $\xi^\pm$ by $\xi^+ D_-\xi^- = D_-\rho$ and $\xi^- D_+\xi^+ = D_+\rho$. The action will be chosen in such a way that the equations of motion for $\Theta^\pm$, $\bar{\Theta}^\pm$ are

$$\bar{D}_-\Theta^- = \bar{D}_+\Theta^+ = D_-\bar{\Theta}^- = D_+\bar{\Theta}^+ = 0\,. \tag{2.7}$$

This implies, in particular, that $\Theta^\pm$ are holomorphic. For any solution of the equations of motion we define X^+ through

$$\begin{aligned} \Theta^+ D_-\Theta^- &= D_-X^+\,, \qquad & \Theta^- D_+\Theta^+ &= D_+X^+\,, \\ \bar{\Theta}^+ \bar{D}_-\bar{\Theta}^- &= \bar{D}_-X^+\,, \qquad & \bar{\Theta}^- \bar{D}_+\bar{\Theta}^+ &= \bar{D}_+X^+\,. \end{aligned} \tag{2.8}$$

Since (2.8) determines X^+ up to a constant shift in terms of $\Theta^\pm$ and $\bar{\Theta}^\pm$, one can treat $\Theta^\pm$ and $\bar{\Theta}^\pm$ as the fundamental target-space variables and treat X^+ as a composite variable. The only subtlety is that on worldsheets with non-zero genus, the equations

$$[D_+, D_-](\Theta^+\Theta^-) = \partial X^+\,, \qquad [\bar{D}_+, \bar{D}_-](\bar{\Theta}^+\bar{\Theta}^-) = \bar{\partial} X^+ \tag{2.9}$$

imply that $(\Theta^\pm, \bar{\Theta}^\pm)$ must satisfy

$$\int dz \int d\kappa^+ \int d\kappa^- \,\Theta^+\Theta^- = \int d\bar{z} \int d\bar{\kappa}^+ \int d\bar{\kappa}^- \,\bar{\Theta}^+\bar{\Theta}^- \tag{2.10}$$

when integrated around a non-contractible loop. In other words, these constraints come from the fact that X^+ defined through (2.8) should be single valued.

To construct a worldsheet action, one needs conjugate momenta for $(\Theta^\pm, \bar{\Theta}^\pm)$ which will be defined as the partially chiral superfields $(W^\pm, \bar{W}^\pm)$ restricted to satisfy

$$\bar{D}_-W^+ = 0\,, \qquad \bar{D}_+W^- = 0\,, \qquad D_-\bar{W}^+ = 0\,, \qquad D_+\bar{W}^- = 0\,. \tag{2.11}$$

With the constraints of (2.3), (2.6) and (2.11), one can construct the critical $N = (2,2)$ superconformally invariant action

$$S = \int d^2z \int d^2\kappa^+ \int d^2\kappa^- \left(X^{+\bar{l}} X^{-l} + W^+\Theta^- + W^-\Theta^+ + \bar{W}^+\bar{\Theta}^- + \bar{W}^-\bar{\Theta}^+ \right), \tag{2.12}$$

which is such that the equations of motion for $W^\pm$, $\bar{W}^\pm$ enforce (2.7). In components (2.12) becomes

$$\begin{aligned} S = \int d^2z \Big(& \partial x^{+\bar{l}} \bar{\partial} x^{-l} + s^{-\bar{l}} \bar{\partial} s^{+l} + \bar{s}^{-\bar{l}} \partial \bar{s}^{+l} + h^{-\bar{l}} h^{+l} + p^+ \bar{\partial}\theta^- + p^- \bar{\partial}\theta^+ + \\ & + \bar{p}^+ \partial\bar{\theta}^- + \bar{p}^- \partial\bar{\theta}^+ + w^+ \bar{\partial}\lambda^- + w^- \bar{\partial}\lambda^+ + \bar{w}^+ \partial\bar{\lambda}^- + \bar{w}^- \partial\bar{\lambda}^+ \Big), \end{aligned} \tag{2.13}$$

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where $X^{+\bar{l}}$ and X^{-l} are defined in (2.5),

$$\begin{aligned} \Theta^{\pm} &= \theta^{\pm} + \kappa^{\pm}\lambda^{\pm} + \cdots, & \bar{\Theta}^{\pm} &= \bar{\theta}^{\pm} + \bar{\kappa}^{\pm}\bar{\lambda}^{\pm} + \cdots, \\ W^{\pm} &= \kappa^{\pm}w^{\pm} + p^{\pm}\kappa^{+}\kappa^{-} + \cdots, & \bar{W}^{\pm} &= \bar{\kappa}^{\pm}\bar{w}^{\pm} + \bar{p}^{\pm}\bar{\kappa}^{+}\bar{\kappa}^{-} + \cdots, \end{aligned} \tag{2.14}$$

and $\cdots$ includes auxiliary fields which have been ignored in the action of (2.13). The left-moving $N=2$ stress tensor for this action is given by the superfield

$$T = D_{+}W^{+}D_{-}\Theta^{-} - D_{-}W^{-}D_{+}\Theta^{+} + D_{+}X^{+\bar{l}}D_{-}X^{-l}, \tag{2.15}$$

which contains critical $N=2$ central charge since $[X^{+\bar{l}}, X^{-l}]$ contribute $c=12$ and $[\Theta^{\pm}, W^{\mp}]$ contribute $c=-6$.

Like the light-cone action of (2.2), the action of (2.12) is manifestly invariant under $\mathrm{SU}(4)\times\mathrm{U}(1)$ Lorentz transformations which transform $[X^{+\bar{l}}, X^{-l}, \Theta^{\pm}, \bar{\Theta}^{\pm}, W^{\pm}, \bar{W}^{\pm}]$ as $[\bar{4}_1, 4_{-1}, 1_{\pm 2}, 1_{\pm 2}, 1_{\pm 2}, 1_{\pm 2}]$ representations where the subscript denotes the U(1) charge. However, in addition to the sixteen manifest light-cone spacetime supersymmetries, the action of (2.12) is also manifestly invariant under four additional spacetime supersymmetries. Under these twenty spacetime supersymmetries parameterized by $[\epsilon^{+l}, \epsilon^{-\bar{l}}, \bar{\epsilon}^{+l}, \bar{\epsilon}^{-\bar{l}}, \epsilon^{\pm}, \bar{\epsilon}^{\pm}]$, the worldsheet variables of (2.12) transform as

$$\begin{aligned} &\delta X^{+\bar{l}} = \epsilon^{-\bar{l}}\Theta^{+} + \bar{\epsilon}^{-\bar{l}}\bar{\Theta}^{+}, \quad \delta X^{-l} = \epsilon^{+l}\Theta^{-} + \bar{\epsilon}^{+l}\bar{\Theta}^{-}, \\ &\delta\Theta^{\pm} = \epsilon^{\pm}, \qquad \delta\bar{\Theta}^{\pm} = \bar{\epsilon}^{\pm}, \\ &\delta W^{+} = -\epsilon^{+l}X^{+\bar{l}}, \qquad \delta W^{-} = -\epsilon^{-\bar{l}}X^{-l}, \qquad \delta\bar{W}^{+} = -\bar{\epsilon}^{+l}X^{+\bar{l}}, \qquad \delta\bar{W}^{-} = -\bar{\epsilon}^{-\bar{l}}X^{-l}. \end{aligned} \tag{2.16}$$

One can check that the supersymmetry transformations of (2.16) anticommute to give translations. The only subtlety is that translations in the x^{+} direction leave the worldsheet variables $\Theta^{\pm}$ and $W^{\pm}$ invariant since (2.8) implies that they are independent of the x^{+} zero mode. And translations in the x^{-} direction transform $\delta W^{\pm} = c\Theta^{\pm}$ and $\delta\bar{W}^{\pm} = c\bar{\Theta}^{\pm}$, as can be seen from the translation generator $P^{+} = \int dz \int d^2\kappa\Theta^{+}\Theta^{-} + \int d\bar{z}\int d^2\bar{\kappa}\bar{\Theta}^{+}\bar{\Theta}^{-}$.

2.2 Consistency of light-cone background

It is clear by construction that the action (2.12) reduces to (2.2) in lightcone gauge. Nevertheless, let us see this more explicitly. The equations of motion for $\Theta^{\pm}$ (2.7) together with the chirality constraints (2.6) imply that $\Theta^{\pm}$ are holomorphic functions which via an $N=2$ superconformal transformation can be set to $\Theta^{\pm} = \kappa^{\pm}$. (2.8) then implies that $\partial x^{+} = 1$.

In a non-flat background, the action of (2.12) is replaced by a non-quadratic action which can depend in a complicated manner on the worldsheet superfields. Since the fermionic $N=2$ superconformal generators are related by a field redefinition to the BRST current and b ghost in the RNS formalism, one expects that quantum $N=(2,2)$ superconformal invariance of the action implies that background is an exact solution of superstring theory. As will now be argued, this condition of $N=2$ superconformal invariance can be used to determine when a given light-cone background in the GS formalism describes a solution of superstring theory.

Suppose one is given an action depending on the light-cone GS variables $X^{+\bar{l}}$ and X^{-l} of (2.3). In general, the action will not be $N=2$ superconformally invariant or even $N=2$ worldsheet supersymmetric. However, by coupling $\Theta^{\pm}$ and $\bar{\Theta}^{\pm}$ in an appropriate manner, one can always construct an action which is classically $N=(2,2)$ superconformally invariant and which reduces to the original action in light-cone gauge where $\lambda^{\pm}=\bar{\lambda}^{\pm}=1$ and $\theta^{\pm}=\bar{\theta}^{\pm}=0$. If this new action is also $N=(2,2)$ superconformally invariant at the quantum level, then the original light-cone GS background describes a solution of superstring theory. Note that an analogous construction exists for bosonic and RNS light-cone backgrounds where, for the bosonic string, ∂x^{+} and $\bar{\partial} x^{+}$ are used to construct conformally invariant actions and, for the RNS superstring, DX^{+} and $\bar{D}X^{+}$ are used to construct $N=(1,1)$ superconformally invariant actions.

Because the parametrization of X^{+} through (2.8) closely resembles the twistor constraints described in [5], the action of (2.12) has been called the $N=2$ twistor-string action. Although this action is not manifestly Lorentz invariant, it can be related to the manifestly Lorentz-invariant "doubly-supersymmetric" action of [6, 3] by introducing additional gauge and auxiliary fields. Furthermore, this doubly-supersymmetric action has been recently related in [9] to the "pure spinor" formalism for the superstring [8] in which the superstring is quantized in a manifestly super-Poincaré invariant manner by constructing a BRST operator out of pure spinors. Also, the U(4)-invariant action of (2.12) can be related to the standard Lorentz-invariant RNS worldsheet action of [15] by bosonizing some of the worldsheet fields and interpreting the resulting theory as an $N=1 \rightarrow N=2$ "embedding" of the RNS superstring [7]. However, none of these Lorentz-invariant descriptions of the superstring preserve manifest $N=2$ worldsheet superconformal invariance. As will be shown in the following section, for Ramond-Ramond plane wave backgrounds in which Lorentz invariance is already broken, the most convenient description is the U(4) formalism which preserves manifest $N=2$ superconformal invariance.

3. U(4) formalism for plane wave background

In an plane wave background, the target-space fields are independent of x^{-} so that the equations of motion for x^{+} are $\partial\bar{\partial}x^{+}=0$. Since x^{-} is contained in the superfields $W^{\pm}$ and $\bar{W}^{\pm}$ in the U(4) formalism, the plane wave background fields should be independent of $W^{\pm}$ and $\bar{W}^{\pm}$. So the most general classically $N=2$ superconformal invariant action for a plane wave background is

$$S=S_0+\int d^2z d^4\kappa U\left(X^{+\bar{l}}, X^{-l}, \Theta^{-}, \Theta^{+}, \bar{\Theta}^{+}, \bar{\Theta}^{-}\right), \tag{3.1}$$

where S_0 is the action in a flat background of (2.12) and U is a general scalar superfield. The left-moving $N=2$ stress tensor in this background is

$$\begin{aligned} T = T_0 &+ D_+X^{+\bar{l}}D_-X^{-m}\partial_{\bar{l}}\partial_m U + D_+X^{+\bar{l}}D_-\Theta^{-}\partial_{\bar{l}}\partial_- U + \\ &+ D_+\Theta^{+}D_-X^{-l}\partial_+\partial_l U + D_+\Theta^{+}D_-\Theta^{-}\partial_+\partial_- U\,, \end{aligned} \tag{3.2}$$

where T_0 is the stress tensor in a flat background of (2.15), $\partial_{\bar{l}}=\partial/\partial X^{+\bar{l}}$, $\partial_l=\partial/\partial X^{-l}$ and $\partial_{\pm}=\partial/\partial\Theta^{\pm}$.

Notice that since $W^\pm$ and $\bar{W}^\pm$ do not appear in the interaction term in (3.1), their equations of motion imply the same equations on $\Theta^\pm$ $\bar{\Theta}^\pm$ that we had in flat space (2.7). These implied, in particular, that we could use an $N=2$ superconformal transformation to set them to $\Theta^\pm = \kappa^\pm$ and $\bar{\Theta}^\pm = \bar{\kappa}^\pm$. So to find which choice of U corresponds to which RR background one can go to lightcone gauge, do the superspace integral in (3.1), and compare with light-cone GS vertex operators. Alternatively, one can use the field redefinition to RNS variables of [7] and compare with the covariant RNS vertex operators of Friedan, Martinec and Shenker [15]. Since Ramond-Ramond vertex operators contain an odd number of unbarred and barred fermions, one finds that

$$\begin{aligned} U = \; & U_{++}(X^{-l}, X^{+\bar{l}})\Theta^+\bar{\Theta}^+ + U_{+-}(X^{+\bar{l}}, X^{-l})\Theta^+\bar{\Theta}^- + \\ & + U_{-+}(X^{+\bar{l}}, X^{-l})\Theta^-\bar{\Theta}^+ + U_{--}(X^{-l}, X^{+\bar{l}})\Theta^-\bar{\Theta}^- \,, \end{aligned} \tag{3.3}$$

describes the RR backgrounds that we are going to be interested in.[2]

Besides the maximally supersymmetric Ramond-Ramond plane wave background of [10], there are other special Ramond-Ramond plane wave backgrounds which preserve less supersymmetries. As shown in [12], these backgrounds are described by either a real harmonic function $V(X^{+\bar{l}}, X^{-l})$ which preserves at least two supersymmetries or a holomorphic function $Y(X^{-l})$ which preserves at least four supersymmetries. For example, the maximally supersymmetric plane wave is described by $Y(X^{-l}) = \delta_{lm} X^{-l} X^{-m}$. In the case of a flat transverse background, it will now be shown how to describe these plane wave backgrounds as exact $N=2$ superconformal field theories using the U(4) formalism. Note that "exact" superconformal invariance will always mean vanishing of the β-function to all perturbative orders in α', and possible non-perturbative contributions will not be discussed here. In [16, 17] an argument was presented for the all order conformal invariance of plane waves with *constant* field strengths. Our argument will also cover certain non-constant field strengths.

By replacing $\int d^2z d^4\kappa \; X^{+\bar{l}} X^{-l}$ with $\int d^2z d^4\kappa \; K(X^{+\bar{l}}, X^{-l})$ in S_0 where $K(X^{+\bar{l}}, X^{-l})$ is a Ricci-flat Kahler potential, the plane wave backgrounds in a flat transverse background are easily generalized to a curved transverse background. In this case, however, the acion is not an exact $N=2$ superconformal field theory because of the usual four-loop divergences in the $N=(2,2)$ non-linear sigma model [18] .

3.1 Plane wave background with real harmonic function

Besides the sixteen light-cone supersymmetries, there are four spacetime supersymmetry transformations that are simply realized in this formalism. As discussed in (2.16), these are generated by spinors that are singlets under SU(4) $\in$ SO(8) and act by shifts as

$$\Theta^\pm \to \Theta^\pm + \epsilon^\pm \,, \qquad \bar{\Theta}^\pm \to \bar{\Theta}^\pm + \bar{\epsilon}^\pm \,, \tag{3.4}$$

where $\epsilon^\pm$ and $\bar{\epsilon}^\pm$ are constant parameters. Although the transformations of (3.4) naively anticommute, one can see from the definition of X^+ in (2.8) that their anticommutator

[2] There are other light-cone RR backgrounds that are described by the vertex operators $\int d^2z \times \int d^4\kappa D_- X^{-[l} \bar{D}_- X^{-j]} \Theta^+\Theta^-\bar{\Theta}^+\bar{\Theta}^- (D_-\Theta^-\bar{D}_-\bar{\Theta}^-)^{-1}$. Since these other RR backgrounds will not preserve any target-space supersymmetry of the type we are considering, we do not consider them any further.

generates a constant shift in X^+. The supersymmetries of (3.4) are generically broken in the plane wave background of (3.1), however, there are special choices of U which preserve either two or four of these symmetries. For example,

$$U = \left(\Theta^- - \Theta^+\right)\left(\bar\Theta^- - \bar\Theta^+\right) V\left(X^{+\bar l}, X^{-l}\right) \tag{3.5}$$

is a Ramond-Ramond plane wave background which is invariant under the two supersymmetries in (3.4) generated by $\epsilon^+ = \epsilon^-$ and $\bar\epsilon^+ = \bar\epsilon^-$. It will now be argued that this supersymmetric background is an exact solution of superstring theory when V is harmonic, i.e. that

$$S = S_0 + \int d^2z d^4\kappa \; \left(\Theta^- - \Theta^+\right)\left(\bar\Theta^- - \bar\Theta^+\right) V\left(X^{+\bar l}, X^{-l}\right) \tag{3.6}$$

is an exact $N=2$ superconformal field theory if $\partial_{\bar l}\partial_l V = 0$.

To prove this, note that since S_0 is free and since the interaction vertex does not involve $[W^\pm, \bar W^\pm]$, the fields $[\Theta^\pm, \bar\Theta^\pm]$ can be set equal to their background values in the interaction term. It is easy to check that Feynman diagrams involving a single interaction vertex are free of divergences if $\partial_{\bar l}\partial_l V = 0$. And Feynman diagrams involving more than one interaction vertex are free of divergences since, by power counting, the only possible divergences could come from terms involving no derivatives on the background variables. But since $[(\Theta^+ - \Theta^-)(\bar\Theta^+ - \bar\Theta^-)]^2 = 0$, there are no such terms.

3.2 Plane wave background with holomorphic function

If $Y(X^{-l})$ is a holomorphic function, the action

$$S = S_0 + \int d^2z d^4\kappa \; \left(Y\left(X^{-l}\right)\Theta^+\bar\Theta^+ + \bar Y\left(X^{+\bar l}\right)\Theta^-\bar\Theta^-\right) \tag{3.7}$$

is no longer invariant under the transformations of (3.4). However, if one defines $W^\pm$ and $\bar W^\pm$ to transform as

$$\begin{aligned} \delta W^+ &= \bar Y\left(X^{+\bar l}\right)\bar\epsilon^- \,, \qquad \delta W^- = Y\left(X^{-l}\right)\bar\epsilon^+ \,, \\ \delta \bar W^+ &= -\bar Y\left(X^{+\bar l}\right)\epsilon^- \,, \qquad \delta \bar W^- = -Y\left(X^{-l}\right)\epsilon^+ \,, \end{aligned} \tag{3.8}$$

the invariance under all four transformations is restored. Note that since $Y(X^{-l})$ is holomorphic, the transformation of (3.8) preserves the constraints of (2.11).

To show that (3.7) is an exact $N=2$ superconformal field theory, first set $[\Theta^\pm, \bar\Theta^\pm]$ to their background values in the interaction vertices. Feynman diagrams involving a single interaction vertex are zero since $Y(X^{-l})$ is holomorphic. For more than one interaction vertex, the only possible divergences come from contractions between $X^{-l}(z_1,\kappa_1)$ and $X^{+\bar l}(z_2,\kappa_2)$ in the vertices $Y(X^{-l})\Theta^+\bar\Theta^+(z_1,\kappa_1)$ and $\bar Y(X^{+\bar l})\Theta^-\bar\Theta^-(z_2,\kappa_2)$. Using standard superspace rules in momentum space [19] , each such contraction is proportional to

$$D_{1+}\bar D_{1+}D_{2-}\bar D_{2-}(\kappa_1-\kappa_2)^2(\bar\kappa_1-\bar\kappa_2)^2 \,. \tag{3.9}$$

Integrating by parts, D_{1+} can be pulled off one of the contractions of (3.9). Since all other contractions are annihilated by D_{1+}, this D_{1+} derivative can only act on the background

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variables. But by power counting, divergences cannot come from terms involving derivatives on the background variables, so the action of (3.7) is an exact $N = 2$ superconformal field theory. Note that if Y had depended on $X^{+\bar{l}}$, this argument would not work since contractions between $X^{+\bar{l}}(z_1, \kappa_1)$ and $X^{-l}(z_2, \kappa_2)$ are not annihilated by D_{1+}.

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A. Green-Schwarz light-cone interaction point operators

To compute scattering amplitudes in light-cone gauge, one needs to introduce light-cone GS operators at the interaction points of the light-cone string diagram [20]. The simplest way to write this operator is

$$|F(z)|^2 = \left|\partial x^{+\bar{l}}\psi^{-l} + \partial x^{-l}\psi^{+\bar{l}}\right|^2 , \tag{A.1}$$

where $(\psi^{-l}, \psi^{+\bar{l}})$ is a fermionic SO(8) vector which is constructed as a spin field from the GS SO(8) spinor $(s^{+l}, s^{-\bar{l}})$. As in the light-cone RNS formalism [21] , this interaction point operator is necessary for preserving SO(9,1) Lorentz invariance and it is easy to see that the GS and RNS light-cone operators are related by SO(8) triality that maps RNS spin fields into GS spinors and maps RNS vectors into GS spin fields.

As is well-known, the RNS light-cone operator $F = \partial x \cdot \psi$ comes from integration over the fermionic $N = 1$ supermoduli for the worldsheet gravitino which couples to the fermionic stress tensor $\partial x \cdot \psi$ [15]. If one describes the light-cone string diagram as an $N = 1$ superconformal map using the coordinates $[\rho(z,\kappa), \xi(z,\kappa)]$ where $[z,\kappa]$ parameterizes the complex $N = 1$ superplane, $N = 1$ superconformal implies that $D_\xi = \partial/\partial\xi + \xi\partial_\rho$ is proportional to $D_\kappa = \partial/\partial\kappa + \kappa\partial_z$, which implies that $\xi = D_\kappa \rho (\partial_z \rho)^{-1/2}$. In this supersheet description, the moduli of the worldsheet gravitino are described by the value of $\xi(z_b)$ at the interaction points z_b where $\partial_z \rho|_{z_b} = 0$. For example, for N-point tree amplitudes with the external vertex operators located at (z_r, κ_r) in the complex superplane,

$$\rho(z,\kappa) = \sum_{r=1}^{N} P_r^+ \log(z - z_r - \kappa\kappa_r) , \qquad \xi(z,\kappa) = \sum_r \frac{P_r^+(\kappa - \kappa_r)}{z - z_r} \left(\sum_r \frac{P_r^+}{z - z_r - \kappa\kappa_r} \right)^{-1/2} ,$$

and the $N - 2$ gravitino moduli $\xi(z_b)$ are proportional to $\sum_r P_r^+ \kappa_r/(z_b - z_r)$ where z_b are the zeros of $\sum_r P_r^+/(z - z_r)$. So integrating over $\xi(z_b)$ has the same effect as introducing light-cone interaction point operators and allows light-cone RNS amplitudes to be expressed as correlation functions on $N = 1$ super-Riemann surfaces [22] .

For example, for tree amplitudes,

$$A = \prod_{b=2}^{N-2} \int d^2(\rho(z_b) - \rho(z_1)) \left\langle \prod_{r=1}^{N} V_r(z_r) \prod_{b=1}^{N-2} |F(z_b)|^2 \right\rangle \tag{A.2}$$

$$= \prod_{b=2}^{N-2} \int d^2(\rho(z_b) - \rho(z_1)) \prod_{b=1}^{N-2} \int d^2\xi(z_b) \left\langle \prod_{r=1}^{N} V_r(z_r, \kappa_r) \right\rangle \tag{A.3}$$

$$= \prod_{r=2}^{N-2} \int d^2 z_r \prod_{r=1}^{N-2} \int d^2\kappa_r |M(z_r, \kappa_r, P_r^+)|^2 \left\langle \prod_{r=1}^{N} V_r(z_r, \kappa_r) \right\rangle, \tag{A.4}$$

where V_r are the light-cone vertex operators and $M(z_r, \kappa_r, P_r^+)$ is an overall measure factor that comes from the jacobian $(\prod_b \partial\rho(z_b)\partial\xi(z_b))/(\prod_r \partial z_r \partial\kappa_r)$ and from the anomalous transformation of the partition function under the superconformal transformation $\rho(z,\kappa)$. Remarkably, one can show that the measure factor simplifies to $M = (z_N - z_1)(z_{N-1} - z_1)$, which is the usual factor that we get in the covariant $N = 1$ formulation [15]. Since (A.4) contains no singularities when interaction points collide, writing the RNS light-cone amplitude in terms of $N = 1$ superconformal correlation functions resolves the problems of light-cone contact terms and simplifies comparison with computations using the standard Lorentz covariant RNS approach. In other words, the transformation from light-cone coordinates $[\rho(z_b), \xi(z_b)]$ to superplane coordinates $[z_r, \kappa_r]$ between (A.3) and (A.4) is valid up to the usual surface terms associated with integration over supermoduli [23]. Including this surface term provides an analytic continuation of the scattering amplitude which automatically includes all light-cone contact terms.

As was shown ten years ago [3], a similar method can be used for treating the GS light-cone operator of (A.1). However, instead of integrating over $N = 1$ worldsheet supermoduli, one needs to integrate over $N = 2$ worldsheet supermoduli. To see this, first write the GS interaction point operator of (A.1) as

$$F(z) = \lim_{z \to z_b} \left(\partial x^{+\bar{l}} s^{+l}(z) \Sigma^-(z_b) + \partial x^{-l} s^{-\bar{l}}(z) \Sigma^+(z_b) \right) (z - z_b)^{1/2} \tag{A.5}$$

where Σ^+ and Σ^- are two components of an SO(8) antichiral spinor constructed as a spin field from the SO(8) chiral spinor $(s^{+l}, s^{-\bar{l}})$. Note that under $SU(4) \times U(1)$, the eight components of an antichiral SO(8) spinor transform as $(6,0)$, $(1,-1)$ and $(1,+1)$ representations, and Σ^- and Σ^+ are defined as the $(1,-1)$ and $(1,+1)$ components. Using the spin field OPE's that $s^{-\bar{l}}(z)\Sigma^+(0) \to z^{-1/2}\psi^{+\bar{l}}$ and $s^{+l}(z)\Sigma^-(0) \to z^{-1/2}\psi^{-l}$, one easily sees that (A.5) is equivalent to (A.1).

Since $\partial x^{+\bar{l}} s^{+l}$ and $\partial x^{-l} s^{-\bar{l}}$ are the $N = 2$ fermionic stress tensors implied by the action of (2.2) and $\Sigma^\pm$ are the $N = 2$ spectral flow operators, one can understand the GS light-cone operator of (A.5) as coming from integration over $N = 2$ supermoduli combined with appropriately chosen U(1) twists. Although the specific combination of $N = 2$ fermionic stress tensors and spectral flow operators appearing in (A.5) might seem strange, it is explained by describing the light-cone string diagram as an $N = 2$ superconformal map from the complex $N = 2$ superplane to the string diagram. Using $[\rho(z, \kappa^+, \kappa^-), \xi^+(z, \kappa^+, \kappa^-), \xi^-(z, \kappa^+, \kappa^-)]$

as this superconformal map where $[z,\kappa^+,\kappa^-]$ parameterize the $N=2$ superplane, $N=2$ superconformal implies that $D_{\xi^\pm}=\partial/(\partial\xi^\pm)+\xi^\mp\partial_\rho$ is proportional to $D_{\kappa^\pm}=\partial/(\partial\kappa^\pm)+\kappa^\mp\partial_z$, which implies that

$$\xi^+=(D_{\kappa^-}\rho)\ (D_{\kappa^-}D_{\kappa^+}\rho)^{-1/2}f(z+\kappa^-\kappa^+)\,,\quad \xi^-=(D_{\kappa^+}\rho)\ (D_{\kappa^+}D_{\kappa^-}\rho)^{-1/2}f^{-1}(z-\kappa^-\kappa^+)\,, \tag{A.6}$$

where $f(z)$ is an arbitrary function associated with U(1) twists.

Note that $\xi^\pm$ must be a periodic function of z in order that the GS fermions $(s^{-\bar{l}},s^{+l})$ are periodic in the string diagram. This means that the function $f(z)$ in (A.6) must be chosen such that it contains square-root cuts at the same locations as the square-root cuts in $(\partial_z\rho)^{\frac{1}{2}}$. So if $\partial_z\rho$ has zeros at $z=z_b$ and poles at $z=z_r$,

$$f=c\sqrt{\prod_b(z-z_b)^{N_b}\prod_r(z-z_r)^{N_r}}\,, \tag{A.7}$$

where c is a constant and (N_b,N_r) are integers. Furthermore, the boundary condition that ξ^+ and ξ^- have at most poles at $z=z_b$ implies that N_b is either ±1. The choice of N_r is fixed by the boundary conditions on the r^{th} external string, e.g. $N_r=1$ implies that the $s^{-\bar{l}}$ zero modes annihilate the "ground state" whereas $N_r=-1$ implies that the s^{+l} zero modes annihilate the "ground state". However, the choice of N_b is unfixed by external boundary conditions, which means that all 2^B possible choices of $N_b=\pm1$ are allowed where B is the number of interaction points. Each such choice corresponds to an individual term in the light-cone operator of (A.5). For example, if $N_b=+1$ for $b=1$ to H and $N_b=-1$ for $b=H+1$ to B, then ξ^+ has poles at z_b for $b=1$ to H and ξ^- has poles at z_b for $b=H+1$ to B. So the term $\prod_{b=1}^H\partial x^{-l}s^{-\bar{l}}\Sigma^+\prod_{b=H+1}^B\partial x^{+\bar{l}}s^{+l}\Sigma^-$ in (A.5) is obtained by integrating over $\prod_{b=1}^H\int d\xi^+(z_b)\prod_{b=H+1}^B\int d\xi^-(z_b)$ where $\xi^\pm(z_b)$ signifies the residue of the pole at $z=z_b$.

As in the light-cone RNS supersheet formalism, this superconformal method allows light-cone GS amplitudes to be expressed as correlation functions on super-Riemann surfaces. For example, for the N-point tree amplitude described by the map $\rho=\sum_{r=1}^N P_r^+\times\log(z-z_r-\kappa^+\kappa_r^- -\kappa^-\kappa_r^+)$,

$$A=\prod_{b=2}^{N-2}\int d^2(\rho(z_b)-\rho(z_1))\left\langle\prod_{r=1}^N V_r(z_r)\prod_{b=1}^{N-2}|F(z_b)|^2\right\rangle \tag{A.8}$$

$$=\prod_{b=2}^{N-2}\int d^2(\rho(z_b)-\rho(z_1))\left|\sum_{K=1}^{2^{N-2}}\prod\int d\xi^+(z_b)\prod\int d\xi^-(z_b)\right|^2\left\langle\prod_{r=1}^N V_r(z_r,\kappa_r^+,\kappa_r^-)\right\rangle \tag{A.9}$$

$$=\prod_{r=2}^{N-2}\int d^2z_r\prod_{b=1}^{N-2}\int d^2\xi^+(z_b)\int d^2\xi^-(z_b)\left|\sum_{K=1}^{2^{N-2}}\prod_{b=1}^H\xi^-(z_b)\prod_{b=H+1}^{N-2}\xi^+(z_b)\right|^2\left\langle\prod_{r=1}^N V_r(z_r,\kappa_r)\right\rangle \tag{A.10}$$

$$=\prod_{r=2}^{N-2}\int d^2z_r\prod_{r=1}^{N-2}\int d^2\kappa_r^+\ d^2\kappa_r^-\ \left|M(z_r,\kappa_r^\pm,P_r^+)\right|^2\left\langle\prod_{r=1}^N V_r(z_r,\kappa_r)\right\rangle, \tag{A.11}$$

where $\sum_{K=1}^{2^{N-2}}$ sums over the 2^{N-2} different possible boundary conditions at the interaction points and $M(z_r, \kappa_r^{\pm}, P_r^{+})$ is an overall measure factor that comes from the jacobian

$$\frac{\prod_b \partial\rho(z_b)\partial\xi^+(z_b)\partial\xi^-(z_b)}{\prod_r \partial z_r \partial\kappa_r^+ \partial\kappa_r^-} \sum_{K=1}^{2^{N-2}} \left(\prod_{b=1}^{H} \xi^-(z_b) \prod_{b=H+1}^{N-2} \xi^+(z_b) \right)$$

and from the anomalous transformation of the partition function under the superconformal transformation $\rho(z, \kappa^+, \kappa^-)$. Unfortunately, unlike the RNS measure factor, the GS measure factor M has a complicated form which has prevented (A.11) from being used to obtain super-Koba-Nielsen-like formulas for GS tree amplitudes. Nevertheless, it can be argued that M has no singularities when interaction points collide. So as in the RNS amplitude of (A.4), expressing the GS amplitude in terms of the $N=2$ superplane coordinates $[z_r, \kappa_r^{\pm}]$ resolves the problem of light-cone contact terms by including a surface term which provides an appropriate analytic continuation of the scattering amplitude.

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